

Std X Science 1

Chapter 1: Gravitation

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*-Textual Questions

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Keys words are underlined

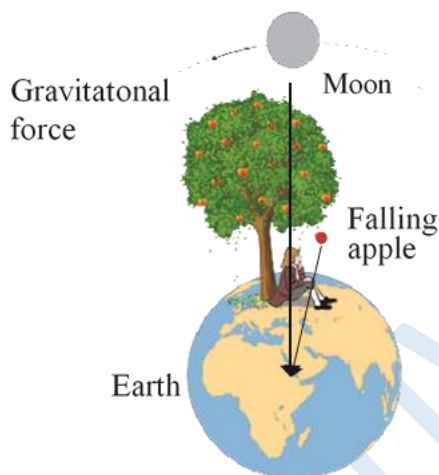
I.) Gravitation

Q.1) What is Gravitation?

Answer: The gravitational force is a universal force and it acts not only between two objects on the earth but also between any two objects in the universe.

Q.2) Explain Gravitation with respect to Sir Isaac Newton?

Answer: the earth attracts the apple towards itself and this attractive force must be directed towards the centre of the earth. The direction from the apple on the tree to the centre of the earth is the vertical direction at the position of the apple and thus, the apple falls vertically downwards.



a.) Force and Motion

**Q.1) What are Newton's Law of Motion?

Answer: Newton's first law of motion states that "an object at rest stays at rest and an object in **motion** stays in **motion** with the same speed and in the same direction unless acted upon by an unbalanced force".

Newton's second law of motion states that "the acceleration of an object is dependent upon two variables - the net force acting upon the object and the mass of the object".

Newton's third law of motion states that "every action has an equal and opposite reaction".

Q.2) Define Force and Motion?

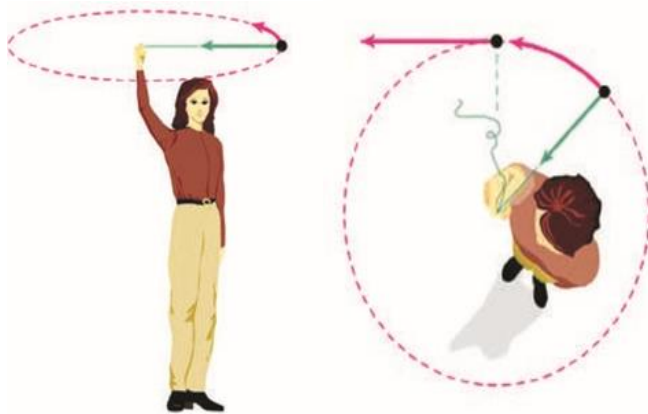
Answer: Strength or energy as an attribute of physical action or movement is called force.

The action or process of moving or being moved is known as motion.

*III.) Centripetal force and Circular motion

Q.1) What is Circular Motion and Centripetal Force?

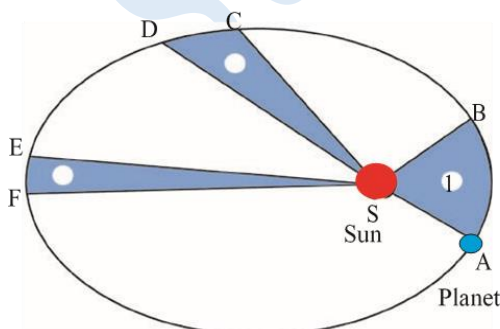
Answer: A Force acts on any object moving along a circle and it is directed towards the centre of the circle. This is called the Centripetal force. 'Centripetal' means centre seeking, i.e. the object tries to go towards the centre of the circle because of this force.



IV.) Kepler's laws

*Q.1) Explain Kepler's laws?

Answer: Kepler's first law: The orbit of a planet is an ellipse with the Sun at one of the foci.



Kepler's second law: The line joining the planet and the Sun sweeps equal areas in equal intervals of time.

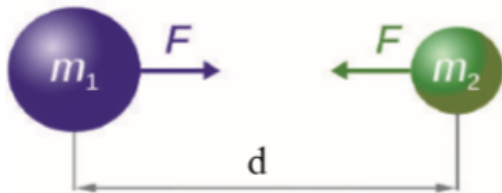
Kepler's third law: The square of its period of revolution around the Sun is directly proportional to the cube of the mean distance of a planet from the Sun.

$$T^2 \propto r^3 \text{ i.e. } \frac{T^2}{r^3} = \text{constant} = K \dots\dots\dots (1)$$

V.) Newton's universal law of gravitation

Q.1) What is Newton's Universal Law of Gravitation?

Answer: Every object in the Universe attracts every other object with a definite force. **This force is directly proportional to the product of the masses of the two objects and is inversely proportional to the square of the distance between them.**



$$F \propto \frac{m_1 m_2}{d^2} \quad \text{or} \quad F = G \frac{m_1 m_2}{d^2} \dots\dots\dots (2)$$

***Q.2) Derive magnitude of Centripetal Force/Uniform Circular Motion?**

Answer: If m is the mass of the object, v is its speed and r is the radius of the circle, then it can be shown that this force is equal to $F = m v^2/r$.

The distance travelled by the planet in one revolution = perimeter of the orbit $2 \pi r$;
 r = distance of the planet from the Sun, Time taken = Period of revolution = T

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{2\pi r}{T}$$

$$F = \frac{mv^2}{r} = \frac{m \left(\frac{2\pi r}{T} \right)^2}{r} = \frac{4 m \pi^2 r}{T^2}, \text{ multiplying and dividing by } r^2 \text{ we get,}$$

$$F = \frac{4 m \pi^2}{r^2} = \left(\frac{r^3}{T^2} \right). \text{ According to Kepler's third law, } \frac{T^2}{r^3} = K$$

$$F = \frac{4 m \pi^2}{r^2 K}, \text{ But } \frac{4 m \pi^2}{K} = \text{Constant}$$

Q.3) Explain Low and High tides?

Answer: High and low tides occur at different times at different places. The level of water in the sea changes because of the gravitational force exerted by the moon. Water directly under the moon gets pulled towards the moon and the level of water there goes up causing high tide at that place. At two places on the earth at 90° from the place of high tide, the level of water is minimum and low tides occur.

VI.) Earth's gravitational force

Q.1) What is Earth's Gravitational Force?

Answer: The earth attracts every object near it towards itself because of the gravitational force. The centre of mass of the earth is situated at its centre, so the gravitational force on any object due to the earth is always directed towards the centre of the earth. Because of this force, an object falls vertically downwards on the earth.

a.) Earth's Gravitational Acceleration

*Q.1) Earth's Gravitational Acceleration?

Answer: The earth exerts gravitational force on objects near it. According to Newton's second law of motion, a force acting on a body

results in its acceleration. Thus, the gravitational force due to the earth on a body results in its acceleration. This is called acceleration due to gravity and is denoted by 'g'.

****Q.2) Derive value of 'g' on the surface of the Earth?**

Answer:

We can calculate the value of g by using Newton's universal law of gravitation for an object of mass m situated at a distance r from the centre of the earth. The law of gravitation gives

$$F = \frac{GMm}{r^2} \dots\dots\dots (3) \quad M \text{ is the mass of the earth.}$$

$$F = mg \dots\dots\dots (4) \quad \text{From (3) and (4), } mg = \frac{GMm}{r^2}$$

$$g = \frac{GM}{r^2} \dots\dots\dots (5) \quad \text{If the object is situated on the surface of the earth, } r = R = \text{Radius of the earth. Thus, the value of g on the surface of the earth is.}$$

$$g = \frac{GM}{R^2} \dots\dots\dots (6) \quad \text{The unit of g in SI units is m/s}^2. \text{ The mass and radius of the earth are } 6 \times 10^{24} \text{ kg and } 6.4 \times 10^6 \text{ m, respectively. Using these in (6)}$$

$$g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} = 9.77 \text{ m/s}^2 \dots\dots\dots (7)$$

b.) Variation in value of g

Q.14) What are the different variations in value of 'g'?

Answer:

A. Change along the surface of the earth: The reason is that the shape of the earth is not exactly spherical and so the distance of a point on the surface of the earth from its centre differs somewhat from place to place. Highest on the poles-9.832m/s² Lowest on the equator-9.78m/s²

B. Change with height: As we go above the earth's surface, the value of r in equation.5 (g=GM/r²) increases and the value of g decreases. However, the decrease is rather small for heights which are small in comparison to the earth's radius.

Place	Height km
g (m/s ²)	

-Surface of the earth average 9.8	0
-Mount Everest 9.8	8.8
-Maximum height reached by manmade balloon 9.77	3.6
-Height of a typical weather satellite 8.7	400
-Height of communication satellite 0.225	35700

C. Change with depth: The value of g also changes if we go inside the earth. The part of the earth which contributes towards the gravitational force felt by the object also decreases. Which means that the value of M to be used in equation.5 ($g=GM/r^2$) also decreases. As a combined result of change in r and M , the value of g decreases as we go deep inside the earth.

c.) Mass and Weight

Q.1) Define Mass and Weight?

Answer: Mass: Mass is the amount of matter present in the object. The SI unit of mass is kg. Mass is a scalar quantity. Its value is same everywhere. Its value does not change even when we go to another planet. According to Newton's first law, it is the measure of the inertia of an object. Higher the mass, higher is the inertia.

Weight: The weight of an object is defined as the force with which the earth attracts the object. The force (F) on an object of mass m on the surface of the earth can be written using equation (4)

$$\text{Weight, } W = F = m g \quad \dots \quad \left(g = \frac{GM}{R^2} \right)$$

Weight being a force, its SI unit is Newton. Also, the weight, being a force, is a vector quantity and its direction is towards the centre of the earth.

Q.2) What are Gravitational Waves?

Answer: Gravitational waves are a very different type of waves. They have been called the waves on the fabric of space-time. Einstein predicted their existence in 1916. These waves are very weak and it is very difficult to detect them. Scientists have constructed extremely sensitive instruments to detect the gravitational waves emitted by astronomical sources. Among these, LIGO (Laser Interferometric Gravitational Wave Observatory) is the prominent one.

VI.) Free Fall

*Q.1) Explain Free Fall and derive its equation?

Answer: Whenever an object moves under the influence of the force of gravity alone, it is said to be falling freely. In free fall, the initial velocity of the object is zero and goes on increasing due to the acceleration due to gravity of the earth. During free fall, the frictional force due to air opposes the motion of the object and a buoyant force also acts on the object. Thus, **true free fall is possible only in vacuum.**

For a freely falling object, the velocity on reaching the earth and the time taken for it can be calculated by using Newton's equations of motion. For free fall, the initial velocity $u = 0$ and the acceleration $a = g$.

$$v = g t$$

$$s = \frac{1}{2} g t^2$$

$$v^2 = 2 g s$$

a.) Gravitational Potential energy

Q.1) What is Gravitational Potential Energy?

Answer: The energy stored in an object because of its position or state is called potential energy. When h is small compared to the radius R of the earth, g to be constant for large values of h , the value of g decreases with increase in h . Object at infinite value of g is zero, value of potential

energy to be zero for smaller distances, the potential energy is less than zero, i.e. it is negative.

VII.) Escape Velocity

*Q.1) What is Escape Velocity?

Answer: The minimum **velocity** that a moving body must have to **escape** from the gravitational field of a celestial body and move outward into space. An object going vertically upwards from the surface of the earth, having an initial velocity equal to the escape velocity, escapes the gravitational force of the earth. The force of gravity, being inversely proportional to the square of the distance, becomes zero only at infinite distance from the earth. This means that for the object to be free from the gravity of the earth, it has to reach infinite distance from the earth. i.e. the object will come to rest at infinite distance and will stay there.

*VIII.) Solved examples

Example 1: Mahindra and Virat are sitting at a distance of 1 metre from each other. Their masses are 75 kg and 80 kg respectively. What is the gravitational force between them?

Given: $r = 1 \text{ m}$, $m_1 = 75 \text{ kg}$, $m_2 = 80 \text{ kg}$ and $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

According to Newton's law

$$F = G \frac{m_1 m_2}{r^2}$$

$$F = 6.67 \times 10^{-11} \times 75 \times 80 / 1^2$$

$$NF = 4.002 \times 10^{-7} \text{ N}$$

The gravitational force between Mahindra and Virat is $4.002 \times 10^{-7} \text{ N}$

This is a very small force. If the force of friction between Mahindra and the bench on which he is sitting is zero, then he will start moving towards Virat under the action of this force. We can calculate his acceleration and velocity by using Newton's laws of motion

Example 1. Calculate the escape velocity on the surface of the moon given the mass and radius of the moon to be 7.34×10^{22} kg and 1.74×10^6 m respectively.

Given: $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$, mass of the moon = $M = 7.34 \times 10^{22}$ kg and radius of the moon = $R = 1.74 \times 10^6$ m.

$$\text{Escape velocity } v = v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

$$\sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.34 \times 10^{22}}{1.74 \times 10^6}}$$

$$= 2.37 \text{ km/s}$$

Escape velocity on the moon 2.37 km/s.

Example 1: If a person weighs 750 N on earth, how much would be his weight on the Moon given that moon's mass is $\frac{1}{81}$ of that of the earth and its radius is $\frac{1}{3.7}$ of that of the earth ?

Given: Weight on earth = 750 N,

Ratio of mass of the earth (M_E) to mass of the moon (M_M) = $\frac{M_E}{M_M} = 81$

Ratio of radius of earth (R_E) to radius of moon (R_M) = $\frac{R_E}{R_M} = 3.7$

Let the mass of the person be m kg

$$\text{Weight on the earth} = mg = 750 = \frac{m G M_E}{R_E^2} \quad \therefore m = \frac{750 R_E^2}{(G M_E)} \dots\dots\dots (i)$$

$$\text{Weight on Moon} = \frac{m G M_M}{R_M^2} \text{ using (i)}$$

$$= \frac{750 R_E^2}{(G M_E)} \times \frac{G M_M}{R_M^2} = 750 \frac{R_E^2}{R_M^2} \times \frac{M_M}{M_E} = 750 \times (3.7)^2 \times \frac{1}{81} = 126.8 \text{ N}$$

The weight on the moon is nearly $1/6^{\text{th}}$ of the weight on the earth. We can write the weight on moon as mg_m (g_m is the acceleration due to gravity on the moon). Thus g_m is $1/6^{\text{th}}$ of the g on the earth.

Example 1. An iron ball of mass 3 kg is released from a height of 125 m and falls freely to the ground. Assuming that the value of g is 10 m/s^2 , calculate

- (i) time taken by the ball to reach the ground
 (ii) velocity of the ball on reaching the ground
 (iii) the height of the ball at half the time it takes to reach the ground.

Given: $m = 3 \text{ kg}$, distance travelled by the ball $s = 125 \text{ m}$, initial velocity of the ball $= u = 0$ and acceleration $a = g = 10 \text{ m/s}^2$.

(i) Newton's second equation of motion gives

$$s = u t + \frac{1}{2} a t^2$$

$$\therefore 125 = 0 t + \frac{1}{2} \times 10 \times t^2 = 5 t^2$$

$$t^2 = \frac{125}{5} = 25 \quad , t = 5 \text{ s}$$

The ball takes 5 seconds to reach the ground.

(ii) According to Newton's first equation of motion final velocity $= v = u + a t$
 $= 0 + 10 \times 5$
 $= 50 \text{ m/s}$

The velocity of the ball on reaching the ground is 50 m/s

(iii) Half time $= t = \frac{5}{2} = 2.5 \text{ s}$

Ball's height at this time $= s$

According to Newton's second equation

$$s = u t + \frac{1}{2} a t^2$$

$$s = 0 + \frac{1}{2} \times 10 \times (2.5)^2 = 31.25 \text{ m.}$$

Thus the height of the ball at half time
 $= 125 - 31.25 = 93.75 \text{ m}$

Example 1: Calculate the gravitational force due to the earth on Mahendra in the earlier example.

Given: Mass of the earth = $m_1 = 6 \times 10^{24}$ kg

Radius of the earth = $R = 6.4 \times 10^6$ m

Mahendra's mass = $m_2 = 75$ kg

$G = 6.67 \times 10^{-11}$ Nm²/kg²

Using the force law, the gravitational force on Mahendra due to earth is given by

This force is 1.83×10^9 times larger than the gravitational force between Mahendra and Virat.

$$F = \frac{G m_1 m_2}{R^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 75 \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} \text{ N} = 733 \text{ N}$$

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